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B.TECH
(SEM I) THEORY EXAMINATION 2022-23
ENGINEERING MATHEMATICS-I

Time: 3 Hours | ***Total Marks: 100***

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 10 = 20

(a) Find the n^{th} derivative of $x.e^{-x}$ at $x = 0$.

(b) Write all the symmetry of $x^2 + y^2 = a^2$

(c) If $u = x + y + z,\ \ v = 2x - y + z,\ \ w = 3x - y + 3z$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

(d) Find the percentage error in the area of an ellipse when an error of $+1$ percent is made in measuring the major and minor axes.

(e) Find the rank of matrix $A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$.

(f) Verify whether the matrix $A = \begin{bmatrix} 4 & 1-3i \\ 1+3i & 7 \end{bmatrix}$ is Hermitian matrix?

(g) Evaluate $\int_2^a \int_2^b \frac{dxdy}{xy}$

(h) Evaluate $\int_0^\infty x^{1/2} e^{-x}\ dx$.

(i) For a scalar field $u = \frac{x^2}{2} + \frac{y^2}{3}$, find the gradient at the point (1, 3).

(j) State Green's theorem.

SECTION B

2. Attempt any *three* of the following: 10x3=30

(a) If $u = \sin^{-1}\left(\frac{x^3+y^3+z^3}{ax+by+cz}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} - 2\tan u = 0$

(b) Expand $f(x, y) = e^x \cos y$as Maclaurin's series up to two degree terms

(c) Show that the system of equations $x + 3y - 2z = 0;\ \ 2x - y + 4z = 0;\ \ x - 11y + 14z = 0$ has a non trivial solution.

(d) Find the volume of solid surrounded by the surface $\left(\frac{x}{a}\right)^{\frac{1}{2}} + \left(\frac{y}{b}\right)^{\frac{1}{2}} + \left(\frac{z}{c}\right)^{\frac{1}{2}} = 1$.

(e) Determine the constants a, b, c, so that $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational.

SECTION C

3. Attempt any *one* part of the following: 10x1=10

(a) If $y = e^{a\sin^{-1}x}$, show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$ and hence calculate $y_n(0)$.

(b) If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, prove that $\frac{1}{2}.\frac{\partial u}{\partial x} + \frac{1}{3}.\frac{\partial u}{\partial y} + \frac{1}{4}.\frac{\partial u}{\partial z} = 0$

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4. **Attempt any *one* part of the following:** **10x1=10**

(a) Divide 24 into three parts such that their product may be maximum.

(b) If $u^3 + v^3 + w^3 = x + y + z,\ u^2 + v^2 + w^2 = x^3 + y^3 + z^3,\ u + v + w = x^2 + y^2 + z^2$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

5. **Attempt any *one* part of the following:** **10x1=10**

(a) Verify Cayley's Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ and hence find A^{-1}.

(b) Find the eigen values & corresponding eigen vectors of the following matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

6. **Attempt any *one* part of the following:** **10x1=10**

(a) Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta \times \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin\theta}} d\theta = \pi$

(b) Evaluate the integral by changing the order of integration $I = \int_0^1 \int_{x^2}^{2-x} xy\ dy\ dx$

7. **Attempt any *one* part of the following:** **10x1=10**

(a) Verify Gauss Divergence theorem for $\vec{F} = (x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + \hat{k}(z^2 - xy)$ taken over the rectangular parallelepiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$

(b) Verify Green's theorem in the plane for $\int_C (x^2 + 2xy)dx + (y^2 + x^3 y)dy$ where C is a square with vertices A(0,0), B(1,0), D(1,1) and E(0,1).

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